

**Mth501 subjective for mid term
Latest spring 2013**

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q1. Applying certain elementary row operation to the elementary matrix

$I_{3 \times 3}$
to produce an identity matrix . [2 marks]

Solution:

Multiply R_2 by $1/5$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We get the identity matrix $I_{3 \times 3}$

Q2. Why is it NOT possible to solve the following system of linear equations applying the Cramer's rule? [2 marks]

$$3x_1 + 2x_2 = 10$$

$$9x_1 + 6x_2 = 30$$

Solution:

$$\text{let } A = \begin{bmatrix} 3 & 2 \\ 9 & 6 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = \begin{bmatrix} 10 \\ 30 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 3 & 2 \\ 9 & 6 \end{vmatrix} = 18 - 18 = 0$$

$$\det(A) = 0$$

since it is a singular matrix and its determinant is zero.
we need determinant of A for applying Cramer's rule

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 0 \end{bmatrix}$$

Q3. Determine whether or not the inverse of the matrix exists? Justify your answer with appropriate reason. [3 marks]

Solution:

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 0 \end{vmatrix}$$

$$|A| = 1(0 - 0) + 2(0 - 6) - 3(0 - 4)$$

$$|A| = 1(0) + 2(-6) - 3(-4)$$

$$|A| = 0 + (-12) + 12$$

$$|A| = 0$$

As the determinant of the given matrix is zero it means it is a singular matrix the inverse of the singular matrix does not exist.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

Q4. Compute $\det(A)$ by using a cofactor expansion across the third row, where

[3 marks]

Solution:

Using cofactor expansion along the first column:

$$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 0 & 2 & 0 \end{vmatrix} = (0)(-1)^{1+1} \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + (2)(-1)^{2+1} \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + (0)(-1)^{3+1} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$$

now if we compare it with the formula

$$\det A = (0)C_{31} + (2)C_{32} + (0)C_{33}$$

$$= (0)(-1)^{1+1} \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + (2)(-1)^{2+1} \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + (0)(-1)^{3+1} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= 0 + (2)(-1)^3 \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + 0$$

$$= 0 + \{-2(6-2)\} + 0$$

$$= 0 - 2(4) + 0$$

$$= -8$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q5. Compute AB using block multiplication, where a

[5 marks]

Solution:

Let

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$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

let

$$A_{11} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, A_{12} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, A_{21} = [0 \ 0], A_{22} = [2]$$

$$B_{11} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, B_{12} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_{21} = [0 \ 0 \ 0], B_{22} = [1]$$

now

$$AB = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

the number of columns of A equals numbers of rows of B

so we can performed multiplication operation:

$$A_{11}B_{11} + A_{12}B_{21} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [0 \ 0 \ 0] = \begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \end{bmatrix}$$

$$A_{11}B_{12} + A_{12}B_{22} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1] = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$$A_{21}B_{11} + A_{22}B_{21} = [0 \ 0] \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + [2][0 \ 0 \ 0] = [0 \ 0 \ 0]$$

$$A_{21}B_{12} + A_{22}B_{22} = [0 \ 0] \begin{bmatrix} 1 \\ 1 \end{bmatrix} + [2][1] = [2]$$

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix} = \begin{bmatrix} 9 & 12 & 15 & 4 \\ 19 & 26 & 33 & 7 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Q6. Consider the vector space $V = R^3$ and the set W consists of all vectors in R^3 whose entries

are equal : that is ,

$$W = \{(a, b, c) : a = b = c\}$$

Show that W is a subspace of R^3 . [5 marks]

Solution:

To check if W is the subspace of R^3 , we 1st check that axiom 1 and 6 of a vector space holds.

Let

$u = (a_1, b_1, c_1)$ and $v = (a_2, b_2, c_2)$ be vectors in W then

$u+v = (a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$ is in W

and also if k is any scalar and $u = (a_1, b_1, c_1)$ in any vector W , then $ku = k(a_1, b_1, c_1)$

Hence W is a subspace.

MIDTERM EXAMINATION
Spring 2013
(MTH501- Linear Algebra (Session - 3))

Q1: If $A = B$, then determine the values of x and y ; where

$$A = \begin{bmatrix} 1 & y + 2 \\ x + 2 & 3 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

Solution:

As we know that matrix $A=B$

its mean that the every entry in $A =$ to the coresponding entry in B

so its clearly seen that value of

$$x+2=4 \dots x=4-2=x=-2$$

$$y+2=2 \dots y=-1$$

Q2: Determine which of the following condition(s) hold(s) for a vector space V over R . Justify your answer with appropriate reason.

$$a) \{ \bar{x} + \bar{y} \mid \bar{x} \in V, \bar{y} \in V \} = V$$

$$b) \{ \bar{x} + \bar{y} \mid \bar{x} \in V, \bar{y} \in V \} = V \times V$$

$$c) \{ \lambda \bar{x} \mid \bar{x} \in V, \lambda \in R \} = R \times V$$

Solution:

(b) and (c) both are true for vector space V over a field R is a set V equipped with an operation called (vector) addition, which takes vectors u and v and produces another vector .

There is also an operation called scalar multiplication, which takes an element and a vector and produces a vector .

Q3: Determine whether or not the solution of the following system of linear equations is possible using inversion algorithm? Justify your answer with appropriate reason.

$$2x_1 + 4x_2 + 3x_3 = 3$$

$$4x_1 + 8x_2 + 6x_3 = 4$$

$$6x_1 + 12x_2 + 9x_3 = 4$$

Solution:

let

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 4 & 8 & 6 \\ 6 & 12 & 9 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}$$

$$\det(A) = 2(72 - 72) - 4(36 - 36) + 3(48 - 48)$$

$$\det(A) = 2(0) - 4(0) + 3(0)$$

$$\det(A) = 0$$

since matrix A is non invertible matrix so we can not apply inversion algorithm here.

Q4: Find a Matrix A such that $W = CA$

$$W = \left\{ \begin{bmatrix} 2b + 2c \\ a + b - 2c \\ 4a + b \\ 3a - b - c \end{bmatrix}; a, b, c \text{ are real} \right\}$$

Solution:

1st we write as a set of linear combinations:

$$w = \left\{ a \begin{bmatrix} 0 \\ 1 \\ 4 \\ 3 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ 1 \\ -1 \end{bmatrix} + c \begin{bmatrix} 2 \\ -2 \\ 0 \\ -1 \end{bmatrix} : a, b, c \text{ in } \mathbb{R} \right\} = \text{span} \left\{ \begin{bmatrix} 0 \\ 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \\ -1 \end{bmatrix} \right\}$$

let

$$A = \begin{bmatrix} 0 & 2 & 2 \\ 1 & 1 & -2 \\ 4 & 1 & 0 \\ 3 & -1 & -1 \end{bmatrix}$$

Q5: Find an LU – decomposition of the Matrix

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

Solution:

$$\begin{aligned} A &= \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} * & 0 \\ * & * \end{bmatrix} \\ &= \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} \text{multiplier} = \frac{1}{2} \begin{bmatrix} 2 & 0 \\ * & * \end{bmatrix} \\ &= \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} \text{multiplier} = -5 \begin{bmatrix} 2 & 0 \\ 5 & * \end{bmatrix} \\ &= \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{bmatrix} \text{multiplier} = -2 \begin{bmatrix} 2 & 0 \\ 5 & 1 \end{bmatrix} \end{aligned}$$

Q6: Consider the vector space $V = \mathbb{R}^2$ and the set W consists of all points in \mathbb{R}^2 such that, $W = \{(a, b) : a, b \geq 0\}$ Show that W is not a subspace of \mathbb{R}^2 .

Solution:

This is not subspace because it is not enclosed under scalar multiplication. So, Vector space $V = \mathbb{R}^2$ where \mathbb{R}^2 not passing through origin is not a subspace of \mathbb{R}^2 .

MIDTERM EXAMINATION
Spring 2013
MTH501- Linear Algebra (Session - 2)

Q1. If $A = \begin{bmatrix} 1 & 3 & | & 2 & 4 \\ 2 & 4 & | & 1 & 3 \\ 5 & 3 & | & 2 & 4 \\ 9 & 8 & | & 6 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 3 & 6 \\ 4 & 5 \end{bmatrix}$ then partition B in such a way that the multiplication can be possible?

Solution:

No as matrix A has 6 partitions it will only able to multiply with matrix B if and only if:
No of columns of A = no of rows of B

Q2. Find the determinant and tell that the given matrix is singular or non singular

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 0 \end{bmatrix}$$

Solution:

$$\text{Det}(A) = 1(0-0) - 2(0-6) + 3(0-4)$$

$$\text{Det}(A) = 1 + 12 - 12$$

$$\text{Det}(A) = 1 \text{ it is non singular matrix}$$

Q3. Determine whether the inverse is possible or not of the given matrix and justify your answer

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 0 \end{bmatrix}$$

Solution:

As

$$\text{Det}(A) = 1(0-0) - 2(0-6) + 3(0-4)$$

$$\text{Det}(A) = 1 + 12 - 12$$

$\text{Det}(A) = 1$ it is non singular matrix

Determinant of the matrix is non singular so its inverse is possible.

Q4. Apply Cramer's rule and find the inverse of $A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$

Solution:

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For the matrix say

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix} \Rightarrow \det A = 10 - (-3) = 13$$

$\Rightarrow A^{-1}$ will also be a 2×2 matrix

As

A_{ji} = submatrix of A formed by deleting row j and column i

So in this case

A_{11} = submatrix of A formed by deleting row 1 and column 1 = $[5]$

A_{12} = submatrix of A formed by deleting row 1 and column 2 = $[-1]$

A_{21} = submatrix of A formed by deleting row 2 and column 1 = $[3]$

A_{22} = submatrix of A formed by deleting row 2 and column 2 = $[2]$

and

$$\det A_i(e_j) = (-1)^{i+j} \det(A_{ji}) = C_{ji}$$

where e_j is the j th column of identity matrix $I_{n \times n}$

So in this case

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$$C_{11} = \det A_1(e_1) = (-1)^{1+1} \det A_{11} = (+1) \det[5] = 5$$

$$C_{12} = \det A_2(e_1) = (-1)^{1+2} \det A_{12} = (-1) \det[-1] = (-1)(-1) = 1$$

$$C_{21} = \det A_1(e_2) = (-1)^{2+1} \det A_{21} = (-1) \det[3] = -3$$

$$C_{22} = \det A_2(e_2) = (-1)^{2+2} \det A_{22} = (+1) \det[2] = 2$$

By Cramer's rule,

$$\{(i, j) - \text{entry of } A^{-1}\} = x_{ij} = \frac{\det A_i(e_j)}{\det A} = \frac{C_{ji}}{\det A}$$

So for the current matrix;

$$\{(1, 1) - \text{entry of } A^{-1}\} = x_{11} = \frac{\det A_1(e_1)}{\det A} = \frac{C_{11}}{\det A} = \frac{5}{13}$$

$$\{(1, 2) - \text{entry of } A^{-1}\} = x_{12} = \frac{\det A_1(e_2)}{\det A} = \frac{C_{21}}{\det A} = \frac{-3}{13}$$

$$\{(2, 1) - \text{entry of } A^{-1}\} = x_{21} = \frac{\det A_2(e_1)}{\det A} = \frac{C_{12}}{\det A} = \frac{1}{13}$$

$$\{(2, 2) - \text{entry of } A^{-1}\} = x_{22} = \frac{\det A_2(e_2)}{\det A} = \frac{C_{22}}{\det A} = \frac{2}{13}$$

Hence by using equation # 4, we get

$$A^{-1} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} \frac{C_{11}}{\det A} & \frac{C_{21}}{\det A} \\ \frac{C_{12}}{\det A} & \frac{C_{22}}{\det A} \end{bmatrix} = \frac{1}{\det A} \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & \frac{-3}{13} \\ \frac{1}{13} & \frac{2}{13} \end{bmatrix}$$

Q5. If $A = \begin{bmatrix} 4 & 5 \\ 6 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -\frac{7}{2} & \frac{5}{2} \\ 3 & -2 \end{bmatrix}$ then show that B is multiplicative of A?

Solution:

$$AB = \begin{bmatrix} 4 & 5 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} -\frac{7}{2} & \frac{5}{2} \\ 3 & -2 \end{bmatrix}$$

$$AB = \begin{bmatrix} \frac{47}{5} & 0 \\ \frac{48}{5} & 3 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

As AB is not equal to I so ,A is not multiplicative identity of B

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